B.Sc. Part-I

Paper-I

Theory of Relativity

Dr. Shiva Kant Mishra
Dept of Physics HDJC

Expt. No. ____________

Some Consequences of The Lorentz Transformation

1.) Loss of Simultaneity :-

If two events are simultaneous for a moving observer in S', The observer measures Their time interval as $\Delta t'$. If The two events happen at The same Position ($\Delta x' = 0$), The Lorentz transformations give $\Delta x = 0$, $\Delta t = 0$ as well. However, if The two events in S' are spatially separate ($\Delta x \neq 0$), we find That for an observer in S.

$\Delta t = \gamma(u)(u/c^2)\Delta x'$, and Therefore the two events are not simultaneous. Things even get worse : Suppose two events A and B happen in S a distance Δx apart, and a time interval Δt after each other. Now if $\Delta x > (c/u)\, c\Delta t$, a moving observer in S' will conclude That $\Delta t' < 0$ which Means That event B happens before event A! fortunately, This does not violate Causality, as a Signal from A to B will at most travel with The Speed of light, which as we will See in The next section, means That ~~of~~ for The conditions given, A and B cannot be causally connected - i.e., you cannot reverse cause and effect, no matter how fast you run.

2.) Time Dilation and Lorentz Contraction :-

A stationary observe in frame S' measures the time difference between two points to be $\Delta t'$ on own clock, while an observer in S will measure the time difference on that clock to be $\Delta t = \gamma(u)\,\Delta t'$. exactly the time dilation result ~~as found in eqn~~. Likewise an observer in S' will measure the length of a stationary stick to be ~~Δt~~ $\Delta L'$. For an observer in S, using a method that reaches the ends of the stick simultaneously (so $\Delta t = 0$), the length is ΔL. we have $\Delta x' = \Delta L' = \gamma(u)\Delta L$. So $\Delta L = \Delta L'/\gamma(u)$, which is the Lorentz contraction result.

3.) Velocity Addition :-

we calculated the speed of an object v as measured in S as a function of the speed v' in S' and the speed u of S'. Substituting the values of the constants we found later, we get the following equation

$$v = \frac{u + v'}{1 + uv'/c^2} \qquad (1)$$

eqn (1) Thus follows directly from the light postulate that is all we used to derive it. It mathematically shows you can never add velocities in such a way as to exceed the speed of light. Setting $u = v' = c$ gives $v = c$, and for any values $u < c$, $v' < c$, you will always get $v < c$.

equⁿ (1) holds for motion in the same direction as the motion of the reference frame. for example if you are on a moving train, and rolling a ball down the length of the train. However, you could also roll the ball in the transverse direction (say y if we call the direction in which the train is moving x). You might think that the observed velocity for the comoving and stationary observers is the same in that case (it is for Galilean transformation), but that's not the case. We have $v_y = dy/dt$, and although dy is invarient, dt is not. Calculating v_y in terms of v'_y (the speed at which the moving observer rolls the ball) is straightforward though, we simply apply the Lorentz transformation to dt.

$$v_y = \frac{dy}{dt} = \frac{dy'}{\gamma(u)\, d\left(t' + \frac{u}{c^2} x'\right)} \quad \text{— (2)}$$

$$= \frac{1}{\gamma(u)} \frac{dy'/dt'}{1 + \frac{u}{c^2} dx'/dt'}$$

$$= \frac{1}{\gamma(u)} \frac{v'_y}{1 + u v'_x / c^2}$$